Econ 7310-1: Monetary Economics 1
Lecture on Inside and Outside Money, etc.

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Money

- Outside Money
- Inside Money

Outside Money: Taxes denominated in money, say dollars

- The static case: see Balasko-Shell article in the McKenzie volume, or simply click on the URL in the reading list
- commodities: $i = 1, \ldots, l$
- agents: $h = 1, \ldots, n$
- consumption: $x_h^i > 0$
- endowment: $\omega_h^i > 0$
- commodity price: $p^i > 0$ with $p^1 = 1$
  commodity price of money: $P^m \geq 0$
- lump-sum tax: τ_h

Equilibrium

- $x_h = (x_h^1, \ldots, x_h^i, \ldots, x_h^l) \in \mathbb{R}^l_{++}$
- $\omega_h = (\omega_h^1, \ldots, \omega_h^i, \ldots, \omega_h^l) \in \mathbb{R}^l_{++}$
- $p = (p, \ldots, p^i, \ldots, p^l) \in \mathbb{R}^l_{++}$
- $P^m \in \mathbb{R}_+$
- $\tau = (\tau_1, \ldots, \tau_h, \ldots, \tau_n) \in \mathbb{R}^n$

Consumer Problem (CP):

$$\begin{aligned} \max_{x_h > 0} \quad & u_h(x_h) \\ \text{s.t.} \quad & p \cdot x_h = p \cdot \omega_h - P^m \tau_h \end{aligned}$$

Competitive Equilibrium

- $(p, P^m) \in \mathbb{R}^l_{++} \times \mathbb{R}_+$ is said to be a *competitive equilibrium* if
  - $\sum_{h=1}^{n} x_h = \sum_{h=1}^{n} \omega_h$
  - x_h solves CP for $h = 1, \dots, n$

The tax vector τ:

- is said to be bonafide, if given $\tau \in \mathbb{R}^n$, there is some equilibrium (p, P^m) with $P^m > 0$.
- is said to be *balanced* if we have

$$\sum_{h=1}^{n} \tau_h = 0$$

Money Taxation:

- If τ is not balanced, then τ is not bonafide
- Hence, τ bonafide $\rightarrow \tau$ balanced
- Proof

$$p \cdot x_h = p \cdot \omega_h - P^m \tau_h$$

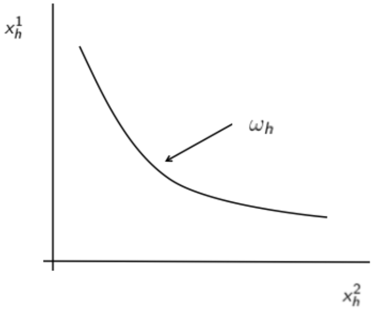
$$p \cdot \sum_h x_h = p \cdot \sum_h \omega_h - P^m \sum_h \tau_h$$

$$P^m \sum_h \tau_h = 0$$

$$P^m = 0 \text{ or } \sum_h \tau_h = 0 \text{ or both}$$

Balanced $\rightarrow$ Bonafide

- True, but not so simple
- Sketch of proof



- Define

$$\begin{aligned}\tilde{\omega}_h &= (\tilde{\omega}_h^1, \tilde{\omega}_h^i) \\ &= (\omega_h^1 - P^m \tau_h, \omega_h^i) \\ &= \text{ tax-adjusted endowment}\end{aligned}$$

The set of equilibrium money prices: $\mathcal{P}^m$

For $l = 1$, $\mathcal{P}^m$ is an interval if τ is bonafide.
Otherwise, $\mathcal{P}^m = \{0\}$.

Worked Examples

- $n = 5, l = 1$

$$\omega = (150, 80, 75, 25, 10)$$

$$\tau = (40, 15, 10, -10, -30)$$

$$\sum_h \tau_h = 25 \neq 0$$

$$\tau \text{ not balanced, } \mathcal{P}^m = \{0\}$$

- Same ω as above

$$\tau = (45, 15, 0, -10, -50)$$

$$\sum_h \tau_h = 0, \tau \text{ balanced}$$

Mr.h's problem is

$$\max u_h(x_h)$$

$$\text{s.t. } x_h + P^m \tau_h = \omega_h$$

$\mathcal{P}^m$ is the set of all P^m such that $x_h > 0$ for $n = 1, \ldots, n$:

$$P^m < \min_h \frac{\omega_h}{\max(0, \tau_h)}$$

For Mr. 1

$$150 - 45P^m > 0, P^m < \frac{150}{45} = 3\frac{1}{3}$$

For Mr.2

$$80 - 15P^m > 0, P^m < \frac{80}{15} = 5.33 > 3\frac{1}{3}$$

Hence

$$\mathcal{P}^m = [0, \bar{P}^m) = [0, 3\frac{1}{3}]$$

No calculations are needed for $h = 3, 4, 5$. Why?

Two Monies

Examples:

- Bi metalism
- Pounds Sterling and Guineas
- Thalers and Pieces of 8
- Etc!

R\$ and B\$, $l = 1, n = 3$
$\tau^R = (2, 1, 0), \tau^B = (5, 3, -12)$

$$\begin{aligned} \max \quad & u_h(x_h) \\ \text{s.t.} \quad & x_h + P^R \tau_h^R + P^B tau_h^B = \omega_h \end{aligned}$$

$$\sum_h x_h + P^R \sum_h \tau_h^R + P^B \sum_h \tau_h^B = \sum_h \omega_h$$

$$P^R \sum_h \tau_h^R + P^B \sum_h \tau_h^B = 0$$

$$e_{RB} = \frac{P^R}{P^B} = -\frac{\sum_h \tau_h^B}{\sum_h \tau_h^R} = -\frac{-4}{3} = \frac{4}{3}$$

Two Monies Continued

The endowment vector is $\omega = (50, 40, 30, 20, 10)$.
Consider a scenario where there are 2 monies, red dollars R and blue dollars B, with respective chocolate prices of money, $P^R \geq 0$ and $P^B \geq 0$.
In each of the following cases, solve for the equilibrium exchange rate between B and R. Do these depend on the endowments ω? Give the economic explanation for your answers.
Solve in each case for the allocation vector as a function of money prices.

Recalling that $x_h = \omega_h - P^R \tau_h^R - P^B \tau_h^B$, we may rearrange the equation to get

$$x_h - \omega_h = -P^R \tau_h^R - P^B \tau_h^B$$

If we sum over h consumers, we get

$$\sum_h (x_h - \omega_h) = -P^m \sum_h \tau_h^R - P^m \sum_h \tau_h^B$$

And since when markets clear, $\sum_h (x_h - \omega_h) = 0$,

$$P^R \sum_h \tau_h^R + P^B \sum_h \tau_h^B = 0 \quad \Rightarrow \quad P^R \sum_h \tau_h^R = -P^B \sum_h \tau_h^B$$

Rearranging further, we get the exchange rate as

$$\frac{P^R}{P^B} = -\frac{\sum_h \tau_h^B}{\sum_h \tau_h^R}$$

Example 1

Suppose $\tau^R = (1, 1, 1, 0, -2)$ and $\tau^B = (1, 0, 0, 0, -2)$
In this case, $\sum_h \tau_h^R = 1 + 1 + 1 - 2 = 1$, while
$\sum_h \tau_h^B = 1 - 2 = -1$, so

$$\frac{P^R}{P^B} = -\left(\frac{-1}{1}\right) = 1$$

Of course, this is also equivalent to $\frac{P^B}{P^R} = 1$ as well.

$$\begin{aligned} x &= (50, 40, 30, 20, 10) - P^R(1, 1, 1, 0, -2) - P^R(1, 0, 0, 0, -2) \\ &= (50 - 2P^r, 40 - P^R, 30 - P^R, 20, 10 + 4P^R) \end{aligned}$$

Example 2

Suppose $\tau^R(1,1,0,-1,-2)$ and $\tau^B = (1,1,1,0,-2)$
Here,

$$\sum_h \tau_h^R = 1 + 1 - 1 - 2 = -1$$

$$\sum_h \tau_h^B = 1 + 1 + 1 - 2 = 1$$

Thus, it again holds that $\frac{P^R}{P^B} = -\left(\frac{-1}{1}\right) = 1$ (and exchanging in the other direction, $\frac{P^B}{P^R} = 1$).

$$\begin{aligned} x &= (50,40,30,20,10) - P^R(1,1,0,-1,-2) - P^R(1,1,1,0,-2) \\ &= (50 - 2P^r, 40 - 2P^R, 30 - P^R, 20 + P^R, 10 + 4P^R) \end{aligned}$$

Example 3

Suppose $\tau^R = (3, 2, 1, 0, -6)$ and $\tau^B = (4, 0, -1, -1, -2)$
We have

$$\sum_h \tau_h^R = 3 + 2 + 1 - 6 = 0$$

$$\sum_h \tau_h^B = 4 - 1 - 1 - 2 = 0$$

The exchange rate is therefore indeterminate, as $\frac{P^R}{P^B} = \frac{0}{0}$ is not well-defined.

The Absence of Money Illusion and Quantity Theory of Money

- Taxes only matter through their real values. Only the term $P^m \tau_h$ matters to Mr. h.
- Absence of money illusion: Let P^m be an equilibrium price of money given the tax vector τ. If the tax vector is multiplied by some scalar λ to become $\lambda\tau$, then $\frac{P^m}{\lambda}$ is *an* equilibrium price of money. In other words, if $\mathcal{P}^m = [0, \bar{P}^m]$ when the tax vector is τ, then when the tax vector is $\lambda\tau$, it follows that $\mathcal{P}^m = \left[0, \frac{\bar{P}^m}{\lambda}\right)$.
- Quantity theory of money: If P^m is an equilibrium price of money when the tax vector is τ, then when taxes become $\lambda\tau$, the equilibrium price of money becomes $\frac{P^m}{\lambda}$.
- The quantity theory of money is true if and only if people believe it to be true, while the absence of money illusion is a statement about sets.

- In other words, if outside money is doubled, then under the quantity theory of money, the price of money will halve (and the price level for real goods, by extension, will double). In contrast, with an absence of money illusion, it is only a *possibility* that the same fiscal policy change will halve the price of money and double the price level. The actual price of money and price level after the tax regime change, however, will be indeterminate.
- Our models are consistent with the AMI, but not strictly with QTM.