Economics 4905
Financial Fragility and the Macroeconomy
Fall 2018
Problem Set 3
Due Monday, October 22, 2018

1. Diamond-Dybvig Bank #1

The probability λ of being impatient is 60%. The utility function is:

$$u(c) = -\frac{1}{c}.$$

The rate of return to the asset harvested late is 200%, i.e.,

$$R = 3$$

(a) What is the depositor's *ex-ante* expected utility W as a function of c_1, consumption in period 1, and c_2, consumption in period 2?

Solution:
$W = \lambda u(c_1) + (1-\lambda)u(c_2)$
$W = \lambda(-\frac{1}{c_1}) + (1-\lambda)(-\frac{1}{c_2}) = 0.35\left(-\frac{1}{c_1}\right) + (1-0.35)\left(-\frac{1}{c_2}\right)$ So

$$W = -\frac{0.35}{c_1} - \frac{0.65}{c_2}$$

(b) Show that the depositor prefers consumption smoothing.

Solution:

$u''(c) < 0$. Hence, $u(c)$ is a strictly concave. A concave functions lies above its chords (Jensen's inequality):
$u(\lambda c_1 + (1-\lambda)c_2) > \lambda u(c_1) + (1-\lambda)u(c_2)$ when $c_1 \neq c_2$,

That is she prefers $(\bar{c}, \bar{c})$ to (c_1, c_2) where $\bar{c} = \lambda c_1 + (1-\lambda)c_2$.

(c) Why can't she insure on the market or self-insure against liquidity shocks?

Solution:

1. Her type is purely her own private information. The insurance company would not trust her to report her type truthfully. She would say that she is impatient even if she is not.

2. She must choose the proportion of liquid assets she holds before she knows her type. The timing does not allow for self-insurance.

Assume that her endowment is 100 and that she deposits her entire endowment in the bank.

(d) What is her utility W in autarky?

Solution:

$W_{autarky} = \lambda u(100) + (1 - \lambda)u(300) = -\frac{17}{3000} = -0.005667$

(e) What is her utility W under perfect smoothing, i.e. when $c_1 = c_2$?

Solution:

$W_{perfect-smoothing} = u(\lambda \cdot 100 + (1 - \lambda) \cdot 300) = -\frac{1}{230} = -0.004348$

(f) What is the bank's resource constraint RC? Write this down precisely. Explain this in words.

Solution:

$$(0.65)(d_2) \leq (\omega - (0.35)d_1)R$$

Or

$$(0.65)(d_2) \leq (100 - (0.35)d_1) \cdot 3$$

Period-2 withdrawals cannot exceed period-2 bank resources. This may be re-written as

$$d_2 \leq \frac{1}{0.65}(100 - 0.35 \cdot d_1) \cdot 3$$

$$d_2 \leq \frac{6000}{13} - \frac{21}{13} \cdot d_1$$

(g) What is the incentive problem? Write this down precisely and explain in words the incentive constraint IC.
Solution:

$d_1 \leq d_2$ (ICC)

If ICC does not hold, everyone will seek to withdraw in period 1.

(h) Find the optimal deposit contract for this bank. What is W if there is no run?
Solution:

$$\arg\max_{d_1,d_2} \lambda u(d_1) + (1-\lambda)u(d_2)$$

subject to RC and ICC.
Using Lagrangian Optimization (see the lecture notes, or Problem 2), we may find that

$$\frac{u'(d_1^*)}{u'(d_2^*)} = R$$

Since $u(c) = -\frac{1}{c}$, it follows that $u'(c) = \frac{d}{dc}\left(-\frac{1}{c}\right) = \frac{1}{c^2} = c^{-2}$,

$$\frac{(d_1^*)^{-2}}{(d_2^*)^{-2}} = \left(\frac{d_1^*}{d_2^*}\right)^{-2} = R \quad \Rightarrow \quad \left(\frac{d_2^*}{d_1^*}\right)^{2} = R$$

And therefore

$$\frac{d_2^*}{d_1^*} = R^{1/2} \quad \Rightarrow \quad d_2^* = d_1^* R^{1/2}$$

Recalling the resource constraint, $d_2^* = \frac{6000}{13} - \frac{21}{13}d_1^*$. Thus,

$$\frac{6000}{13} - \frac{21}{13}d_1^* = d_1^* \cdot 3^{1/2} \quad \Rightarrow d_1^*(\frac{21}{13} + 3^{1/2}) = 1000$$

So

$$d_1^* = \frac{6000/13}{(21/13 + 3^{1/2})} = 137.88$$

$$d_2^* = \frac{6000}{13} - \frac{21}{13} \cdot 137.88 = 238.81$$

As such, $d_1^* = 137.88, d_2^* = 238.81$
$W_{no-run} = 0.35u(d_1^*) + 0.65u(d_2^*) = -0.005260$

(i) Why is there a run equilibrium for this bank?
Solution:

$d_1^* = 137.88 > 100$

If every depositor attempted to withdraw at once (not just the impatient ones, but the impatient ones, too), then the bank will not be able to pay everyone at once

(j) Calculate the following numerical values of *ex-ante* utility W and rank them in numerical ascending order: $W_{autarky}$, $W_{perfect\ smoothing}$, W_{no-run}, W_{run}.
Solution:
$W_{run} = (100/d_1^*)u(d_1^*) = -0.005260$

$W_{autarky} < W_{no-run} = W_{run} < W_{perfectsmoothing}$

(k) Assume that the run probability s is 10%. Will individuals deposit in this bank? That is, will they accept this banking contract? Explain.
Solution:

$(0.1)W_{run} + (1 - 0.1)W_{no-run} = (0.1)(-0.005260) + (0.9)(-0.005260) = -0.005260 > -0.005667 = W_{autarky}$

Consumers will deposit at the bank if there is a 10% run probability.