Economics 4905
Financial Fragility and the Macroeconomy
Fall 2018
Ungraded Problem Set

1 Consumer Problem

Consider

$$\max_{x_h^1, x_h^2} \quad u_h(x_h^1, x_h^2) = A \log x_h^1 + B \log x_h^2$$
$$\text{subject to} \quad p^1 x_h^1 + p^2 x_h^2 = p^1 \omega_h^1 + p^2 \omega_h^2 = w_h$$

Solution:

1. **By substitution:**
   Rearranging the budget constraint gives
   $$x_h^2 = \frac{w_h - p^1 x_h^1}{p^2}$$
   The consumer problem becomes
   $$\max_{x_h^1} \quad A \log x_h^1 + B \log \left(\frac{w_h - p^1 x_h^1}{p^2} \right)$$
   Taking the first-order derivative with respect to x_h^1 and equating it to zero gives
   $$\frac{A}{x_h^1} - \frac{Bp^1}{p^2} \frac{p^2}{w_h - p^1 x_h^1} = 0$$
   Solving for x_h^1
   $$x_h^1 = \frac{A}{A+B} \frac{w_h}{p^1}$$
   Plugging this into the budget constraint to solve for x_h^2
   $$x_h^2 = \frac{B}{A+B} \frac{w_h}{p^2}$$

2. **By Lagrangian:**
   Setting up the Lagrangian of the problem:
   $$\mathcal{L} = A \log x_h^1 + B \log x_h^2 + \lambda (w_h - p^1 x_h^1 - p^2 x_h^2)$$
   Taking the derivative of the Lagrangian with respect to x_h^1, x_h^2 and λ and set them to zero:
   $$\frac{\partial \mathcal{L}}{\partial x_h^1} = \frac{A}{x_h^1} - \lambda p^1 = 0$$
   $$\frac{\partial \mathcal{L}}{\partial x_h^2} = \frac{B}{x_h^2} - \lambda p^2 = 0$$
   $$\frac{\partial \mathcal{L}}{\partial \lambda} = w_h - p^1 x_h^1 - p^2 x_h^2 = 0$$

The solution to the above system of equations is

$$x_h^1 = \frac{A}{A+B}\frac{w_h}{p^1} \qquad x_h^2 = \frac{B}{A+B}\frac{w_h}{p^2} \qquad \lambda = \frac{A+B}{w_h}$$